**Predicate Logic**

* **Proof theory – natural deduction**
* **Forall elimination**
  + i.e. if a formula is true for all values, then it is true for any particular value
  + ∀x . P → P[t/x]
  + In the substitution, no free variables in t should be captured
  + Substitution – P[t/x] “t substituted for x” where t is any term
    - P[t/x] is the formula obtained by replacing every free occurrence of x in P with t
  + Variable capture – a free variable becomes bound after substitution
    - E.g. given P[t/z], if there is free variable(s) w in t, and;
    - There is free z within the scope of ∀w or ∃w, then w would be captured through substitution – substitution is not allowed
  + “t is free for z in P” – in P, no free z’s occur within the scope of ∀w or ∃w for any free w in t
* **Exists introduction**
  + i.e. if a formula is true for some value, then there exists a value for which the formula is true
  + P[t/x] → ∃x . P
  + Choose a term t in a formula (already in the proof) → ∃x . formula with t replaced with x
  + Not all occurrences of t need to be replaced
  + t must have no bound variables in it
  + Choose an x that has not been used in the proof
* **Genuine variable** – free variable such that its universal quantification yields a formula that is true
  + Represents any value
  + E.g. ∀xg . xg + xg = 2xg
* **Unknown variable** – free variable such that its existential quantification yields a formula that is true
  + Represents a specific value
  + E.g. ∃xu . xu + 1 = 2
* **Forall introduction**
  + i.e. if a formula is true for an arbitrary value then it is true for all values
  + for every xg {

…

P[xg/x]

}

∀x . P

* + xg must only be used within the subproof and have not been used previously in the proof
  + xg is a specific value and cannot be arbitrarily replaced
* **Exists elimination**
  + i.e. if a formula P is true for some value, derive (using xu) a formula Q holds that doesn’t contain xu; then conclude Q
  + ∃x . P

for some xu P[xu/x] {

…

Q

}

Q

* + xu must only be used within the subproof and have not been used previously in the proof
* **Proof theory – semantic tableaux**
* Universal quantification/instantiation
  + ∀x . P(x) → P(t) where t is free for x in P
* Existential quantification/instantiation
  + ∃x . P(x) → P(y) where y has not been used in the tableaux so far
* Negative universal quantification
  + ¬(∀x . P(x)) → ∃x . ¬P(x)
* Negative existential quantification
  + ¬(∃x . P(x)) → ∀x . ¬P(x)